

Large space shell model calculations with small space results

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Abstract

We note that in large space shell model calculations and experiment one sometimes get results, the form of which also appear in smaller space calculations. On the other hand there are some results which demand the large space approach.

1 Introduction

Let us start by saying that large shell model calculations are absolutely essential for getting quantitative results for comparison with experiment. And indeed model spaces are getting larger and larger. Three body interactions are being included and ab initio calculations are being performed. If one examines the wave functions of these large space calculations one finds very little of the so called leading configurations - the ones that were used in the early days. One might expect that by this time the whole shell model would be destroyed yet somehow this is not the case, and this is what we will here address this problem.

2 Quadrupole moments and magnetic moments

We start with experiments and calculations that were performed for the semi-magic $N=28$ isotones by Speidel et al. [1]. The focus was on magnetic moments of excited $2+$ states of even-even nuclei but the ground states even=odd were also included in the analysis. There is a theorem that in the single j shell of particles of one kind all g factors are the same. we are here dealing with close shell of neutrons and valence protons in the $f_{7/2}$ shell. In the simple Schmidt model the values of the g factors would be $(3 + 2.793)/3.5 = +1.655$. However as seen in ref[1] the overall values are less than this and are not constant. It was long ago pointed out by Arima, Horie and Noya [2][3][4] in a first order perturbation calculation that the g factors would be quenched. Not only that but the quenching would be n dependant. The more protons in the $f_{7/2}$ shell the more quenching. The experimental values for both the even=even and even odd nuclei show this behaviour. Furthermore large space calculations beyond first order perturbation theory, although giving quantitatively somewhat different values for the g factors, more or less preserve the overall picture of g factors lying on a straight line with a negative slope. In T. Ohtsubo et al. [5] the missing ^{49}Sc moment is measured with a value of 5.615(35). A theoretical analysis which is more or less identical to the one made previously by Speidel et al. [1] was also performed for the even-odd nuclei only, however with theoretically obtained renormalized magnetic moment operators,

As another example we discuss quadrupole moments of ground states of even odd nuclei in the Calcium isotopes. In a recent publication R.F. Garcia [6] presented in part measurements of this for ^{43}Ca , ^{45}Ca and ^{47}Ca (also outside the $f_{7/2}$ shell ^{49}Ca and ^{51}Ca). In their Fig 4 they show that results. The ground state quadrupole moments appear to lie roughly on a straight line starting from negative at ^{41}Ca to positive beyond ^{44}Ca . We here point out that there is a well known formula which gives this behaviour.

$$Q = -(2j + 1 - 2n)/(2j + 2) \langle r^2 \rangle e_{eff} \quad (1)$$

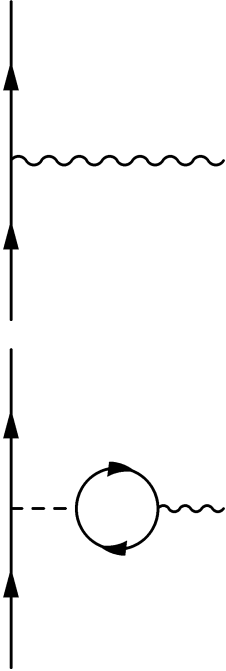
This appears in Lawson's book [7] and was used by Robinson et al. in a study of the Ge isotopes [8]. The results are also consistent with the statement that in a single j shell of particles of one kind $Q(hole) = -Q(particle)$. The $NN + 3N$ calculations in the Garcia et al. [6] paper follow this trend although in detail there are some deviations. The Q values they give for $A = 43, 45$ and 47 are $-0.0246, +0.0252$ and $+0.0856$.

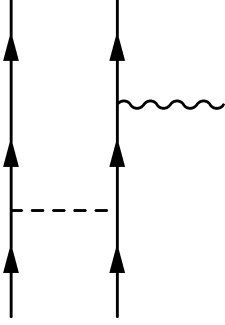
When they come to ^{49}Ca in the single j shell we have one $p_{3/2}$ particle so we expect a negative Q whilst for ^{51}Ca which is a $p_{3/2}$ hole a positive Q . This obtained in both the experiment and $NN + 3N$ calculation [6].

In this paper they also discuss magnetic moments of the even odd ground states. The magnetic moments in the $NN + 3N$ calculation appear to lie on a straight line reminiscent of the first order calculations of Arima et al. [2][3][4]. The calculated values for $A = 43, 45$ and 47 in the $NN + 3N$ calculation are $-1.56, -1.45$ and -1.38 respectively. The experiment shows a somewhat flatter behavior.

We now come to cases where there are major disagreements between experiment and the single j shell models and where large space calculations are an absolute necessity. These concern the magnetic moments of excited states of the Ca isotopes, $s J = 2^+$ states. Whereas the g factor of an $f_{7/2}$ neutron is negative $g = -1.193/3.5 = -0.547$, the measured values for the 2^+ states in the work of Speidel et al. [9] are positive. We have a gross violation of the single j shell model. In that work one put in by hand about a 50% admixture of highly deformed states with $g = Z/A$ which is about 0.5. To round things out a bit a 50% admixture of single j with a g factor of -0.5 and a deformed mixture of $+0.5$ leads to an overall g factor of zero. It remains a challenge to see if the ab initio calculations can handle these highly deformed admixtures. In a work of Taylor et al. [10] it is noted that the g factor of ^{46}Ca returns to negative although still far from the Schmidt model. This is an indication that ^{48}Ca is a better closed shell than ^{40}Ca .

The extreme non-perturbative behaviours for the 2^+ states in ^{42}Ca and ^{44}Ca suggest that deformed admixtures could be of importance also for the previously mentioned ground states of the even-odd Ca isotopes. We suggest that the flat behaviour for the even-odd Ca isotopes rather than the downslope shown for the $N = 28$ isotones by Speidel et al. [1] could be due to these non-perturbative admixtures. As an example in ^{41}Ca the Schmidt magnetic moment is $-1.913\mu_N$ but experiment is $-1.58\mu_N$. In second order perturbation theory Mavromatis et al. gets close to this result [11]. However the inclusion of meson exchange currents gets one back to square zero, very close to the original Schmidt value. See for example the review by I.A. Towner [12]. One can get back to the experimental result by including about 15% of the highly deformed admixture.





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